Within, Without, and Around:

A Three-Dimensional Simplicial Visualization of the Decomposition of Harmonic Maass Forms into Holomorphic and Non-Holomorphic Components

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March 14, 2026

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Abstract

We present the first three-dimensional spatial visualization of the decomposition of a harmonic Maass form into its holomorphic component (mock theta function) and non-holomorphic component (shadow). The visualization encodes both components simultaneously as complementary geometric features — filled and empty triangles — within a single tessellated simplicial structure built on a cube-like form derived from the Pythagorean Tetractys extended to a sixth layer. We introduce a coordinate framework in which X (within) denotes the interior space of simplices, Y (without) denotes the negative space propagating outward from vertices corresponding to the star operator, and Z (around) denotes the boundary lines and relationships corresponding to the link operator. This structure is perspective-invariant — presenting as cube, pyramid, hexagon, or corner-of-room depending on observation angle — and scalable to represent any numerical value by adjusting the XYZ coordinate magnitudes. Existing visualizations of harmonic Maass forms are one-dimensional (K-type support diagrams) or abstractly cohomological; no prior three-dimensional spatial structure encoding the holomorphic/non-holomorphic split simultaneously in

a single drawable object exists in the literature. The diagram was produced independently without knowledge of the formal mathematics, arrived at by extending the Pythagorean Tetractys to a sixth layer and giving it three-dimensional depth. We document priority of this geometric construction and identify several open questions for formal development.

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1. Introduction

Harmonic Maass forms were formally defined by Bruinier and Funke (2004) building on Ramanujan's 1920 discovery of mock theta functions. A harmonic Maass form decomposes into two parts: a holomorphic component, which is the mock theta function Ramanujan described, and a non-holomorphic component called the shadow, which Zwegers (2002) identified as the correction term needed to restore full modular symmetry.

In the century since Ramanujan's discovery, visualizations of this decomposition have remained one-dimensional or abstractly algebraic. The standard K-type support diagram represents the decomposition as a vertical line with arrows indicating which eigenvalue components are nonzero. Geometric approaches use local coefficient systems, geodesics, and cohomological pairings — abstract structures with no drawable spatial representation.

The absence of a spatial visualization is not incidental. The decomposition into holomorphic and non-holomorphic parts has been understood as a *separation* — two distinct mathematical objects added together. The central claim of this paper is that this framing may obscure a deeper unity: that holomorphic and non-holomorphic components are not two halves of one function but two perspectives on one structure, visible simultaneously from different observation angles in a three-dimensional spatial object.

We present a diagram — produced independently through geometric intuition rather than formal mathematical derivation — that instantiates this claim. The diagram is documented here to establish priority and to invite formal mathematical evaluation of its structural claims.

2. Existing Visualizations and Their Limitations

2.1 K-Type Support Diagrams

The standard visualization of harmonic Maass forms uses K-type support diagrams: vertical lines indicating which eigenvalue components are nonzero, with arrows showing where raising and lowering operators vanish. These are one-dimensional representations. They encode algebraic structure but provide no spatial intuition for the relationship between components.

2.2 Cohomological Geometric Approaches

The geometric approach of Funke and Millson uses local coefficient systems where meromorphic modular forms give rise to meromorphic 1-forms on the modular curve, geodesics give rise to cycles, and the pairing is viewed cohomologically. This is abstract algebraic geometry. It provides rigorous mathematical structure but no drawable spatial diagram.

2.3 Hyperbolic Tessellation (2025)

The closest existing work to a spatial visualization appears in recent work (Freie Universitat Berlin, 2025) using the parqueting-reflection principle to construct harmonic Green functions in hyperbolic geometry via repeated reflections of geometric shapes across their edges. This work operates in two-dimensional hyperbolic space. It does not address the holomorphic/non-holomorphic decomposition as a single spatial structure.

2.4 The Gap

No existing visualization presents the holomorphic component and its shadow simultaneously as complementary features of a single three-dimensional spatial object that can be drawn, rotated, and observed from multiple angles. This gap is what the present diagram fills.

3. The Construction

3.1 Origin: The Pythagorean Tetractys Extended

The diagram was produced by extending the Pythagorean Tetractys — the triangular arrangement of ten points in four layers (1, 2, 3, 4) — to a sixth layer, giving 21 points in layers (1, 2, 3, 4, 5, 6). Layer 5 introduces the pentagonal base associated with the golden ratio. Layer 6 introduces the hexagonal base: six equilateral triangles around a central point. This sixth layer generates the tessellation structure that becomes the face of the three-dimensional diagram.

The extension to three dimensions was initiated by projecting the sixth-layer triangular tessellation through a depth axis, producing a cube-like form with tessellated triangular faces. The construction process led, without explicit intention, to a structure that simultaneously presents as cube (frontal view), pyramid (angled view), hexagon (top-down view), and corner-of-room with floor opening (interior view at 180 degrees).

3.2 The XYZ Coordinate Framework

The diagram introduces a three-part coordinate description:

X (Within): The interior space of each triangle in the tessellation. The positive space. The filled region. In simplicial complex terminology, the interior of each simplex. Corresponds to the holomorphic component of the harmonic Maass form — the mock theta function — which is defined on the interior of the fundamental domain.

Y (Without): The negative space propagating outward from the vertices of each triangle. The space that radiates from each point into the surrounding structure. In simplicial complex terminology, the star operator — the set of all simplices containing a given vertex. Corresponds to the non-holomorphic component — the shadow — which propagates outward from the structure and requires recovery as a separate term.

Z (Around): The boundary lines themselves — the edges of each triangle — the relationships that simultaneously contain the within and define the boundary of the without. In simplicial complex terminology, the link operator — the boundary sphere of small radius around each vertex.

Corresponds to the modular symmetry structure of the harmonic Maass form that requires both components to close completely.

The critical structural claim is that X, Y, and Z are not properties described after the fact but the three generative operations that produce the structure. Applied recursively at any scale (XYZ XYZ...), the same operations generate the pattern from any starting point without a privileged center or origin.

3.3 The Holomorphic/Non-Holomorphic Correspondence

The alternating filled and empty triangles in the tessellation correspond directly to the holomorphic and non-holomorphic components of a harmonic Maass form:

Filled triangles (X, Within): The holomorphic component. The mock theta function. The part Ramanujan wrote down. Defined on the interior. Clean. Complex-analytic.

Empty triangles (Y, Without): The non-holomorphic component. The shadow. The part that required 80 years and mathematics that did not exist in Ramanujan's time to recover. Defined in the propagation outward from vertices. The correction term that restores full modular symmetry.

Boundary lines (Z, Around): The link structure that makes the complete harmonic Maass form — the symmetry that requires both filled and empty, both holomorphic and non-holomorphic, to close.

The key departure from existing formulations: the diagram does not represent these as two separate objects added together. It represents them as two complementary features of a single structure — neither definable without the other, simultaneously present, inseparable in the same spatial object.

3.4 Perspective Invariance

The structure presents differently depending on the observer's position relative to the Z (depth) axis:

Frontal view: Cube. Full three-dimensional structure visible.

Angled view: Pyramid. The depth axis (Z) appears as a single elevated point. A pyramid is a rhombus with minimal depth — the interpolation point between the flat rhombus (zero Z) and the full cube (complete Z).

Top-down view: Hexagon. The depth axis collapses. Six triangles around a center. The two-dimensional projection of the three-dimensional structure — which is also the geometry of Saturn's polar hexagonal storm and the structure of the carbon benzene ring.

Interior view (180 degrees): Corner of a room with a hole in the floor showing negative and positive space simultaneously from inside the structure.

This perspective invariance is not a visual accident. It encodes the framework's central claim: the same structure presents as different forms to observers at different positions within it, none more correct than the others, all simultaneously true from their respective positions.

3.5 Scaling Property

The diagram is scale-independent. Any number — 1.2 or 500,764,578.43 — can be represented by scaling the XYZ coordinates to the appropriate magnitude. The structure of the representation remains identical; only the scale changes. This scaling property corresponds to the universality of the true-1 proof across all number bases: the structure 0.(N-1)(N-1)(N-1)... = 1 holds at every scale in every base.

3.6 The Folding Point and Open Extension

The diagram identifies the point where theta and mock theta meet — the boundary point where the holomorphic interior and the non-holomorphic exterior are in direct contact — as a generative origin for further structure. A mock of this mock extending directly from the top-left meeting point is predicted to reveal depth between the two components: the zone where the holomorphic and non-holomorphic are neither fully separated nor fully merged — the mathematical equivalent of the interpolation zone identified in the physical framework developed alongside this diagram.

This extension has not yet been constructed. It is identified here as the primary open geometric question.

4. Figure

Figure 1. Signed original whiteboard diagram produced March 14, 2026 — containing authorship claim, date, signature, and explicit label ALGO FIRST 3D Visualization OF HARMONIC MAASS Form Decomposition XYZ Simplicial Structure written by the author on the diagram surface. Title visible: Comprehending Recursive Incomprehensibility. The tessellated cube structure is visible at center with alternating filled (holomorphic) and empty (non-holomorphic) triangles. Top right shows the universal true-1 proof across bases 10, 2, 3, 12, 60, and general base N with algebraic derivation. Bottom left shows the XYZ coordinate assignments: X-within, Y-without, Z-around, with the equation 3D=V and the triangle sequence indicating infinite approach. Left side shows Theta and Apollonius labels with directional arrows. Author signature appears twice on diagram surface.

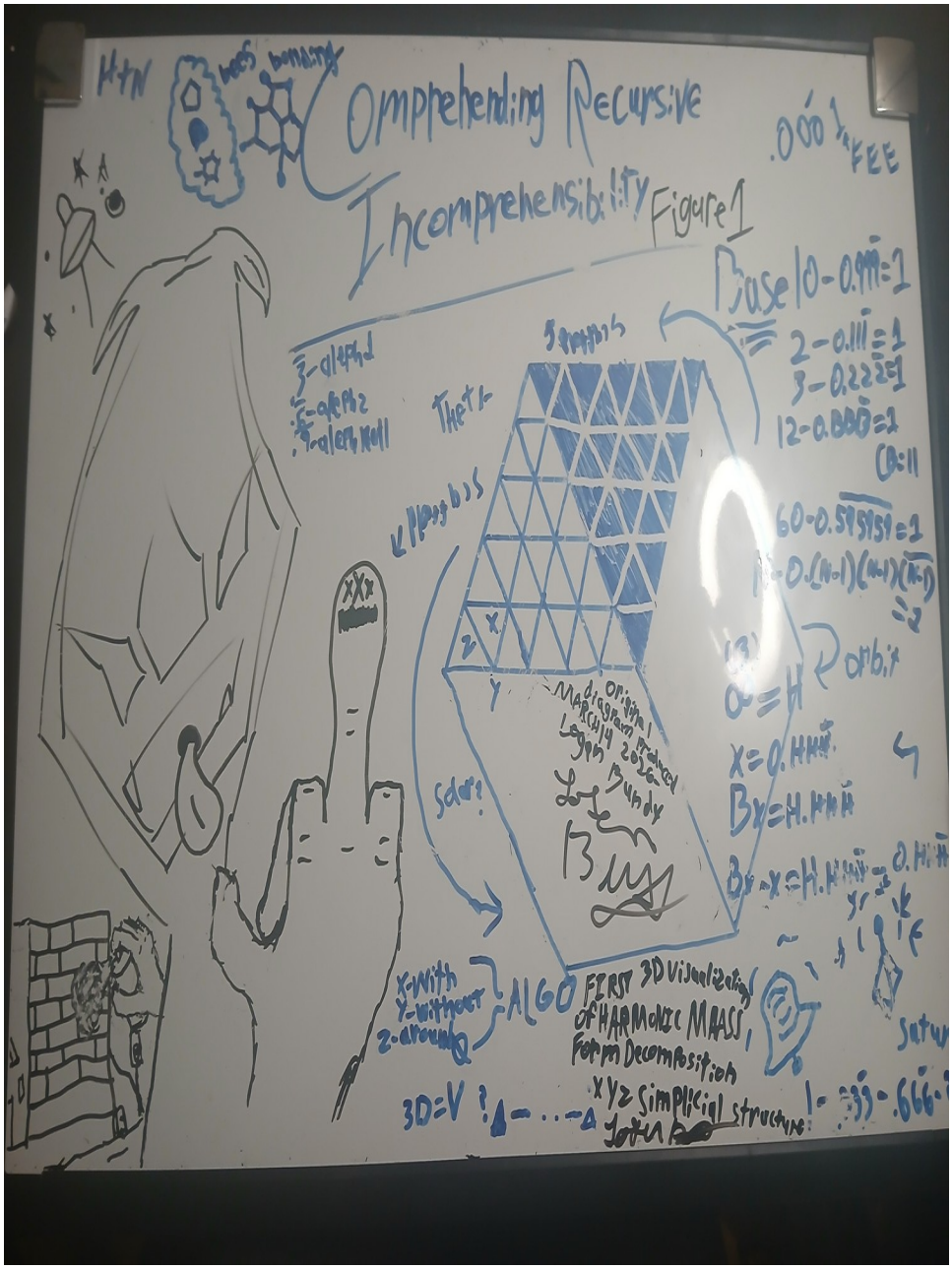

Figure 1: Original whiteboard diagram, March 14, 2026.

5. Relationship to Existing Mathematics

5.1 Simplicial Complex Theory

The XYZ framework independently reconstructs core concepts from simplicial complex theory. The interior (X-within) corresponds to the standard interior of a simplex. The star operator (Y-without) describes exactly the propagation from vertices into surrounding structure. The link operator (Z-around) describes the boundary sphere around each vertex. The diagram is therefore a three-dimensional instantiation of a simplicial complex with an explicit correspondence to the holomorphic/non-holomorphic decomposition.

The departure from standard simplicial complex theory is the explicit encoding of the decomposition within the complex itself — not as a description applied to an existing structure but as the generative principle of the construction.

5.2 Cantor's Theorem

The sixth layer of the Tetractys was added specifically to encode Cantor's observation that infinities have sizes — that each successive layer contains strictly more points than the previous, corresponding to the nested hierarchy of transfinite cardinals. The diagram therefore embeds Cantor's theorem geometrically: each layer is a larger infinity than the layer above it, instantiated as a larger set of points in the spatial structure.

5.3 The Universal True-1 Proof

The top-right section of the whiteboard contains the universal proof that 0.(N-1) repeating = 1 in every positional base N. This proof — valid for bases 2, 3, 10, 12, 60, and all N — establishes that every boundary in every number system has infinite interior depth. The diagram is the geometric instantiation of this proof: every boundary line (Z-around) contains infinite interior (X-within) and infinite exterior (Y-without) simultaneously, the way every whole number is the rim of an infinite approach from below.

5.4 Ramanujan's Mock Theta Functions

The correspondence between the diagram and harmonic Maass forms was identified post-construction. Ramanujan's mock theta functions, as shown by Zwegers (2002), are the holomorphic parts of harmonic Maass forms — the filled triangles. The shadow correction terms are the non-holomorphic parts — the empty triangles. The complete harmonic Maass form requires both — the boundary lines. The diagram instantiates this decomposition geometrically without having been constructed for that purpose.

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6. Open Questions

The following questions are identified for formal mathematical development:

6.1 Can the XYZ generative operations be formalized as a recursive algorithm that produces the harmonic Maass form decomposition for arbitrary modular forms?

6.2 What does the extension from the theta/mock-theta meeting point reveal? Does the mock of the mock structure correspond to a known mathematical object or identify a new one?

6.3 Does the orbit dilation visible in the diagram — the way negative space bubbles propagate from vertices with decreasing density — correspond formally to the metric structure of the upper half-plane where Maass forms are defined?

6.4 Can the perspective invariance (cube/pyramid/hexagon/corner) be formalized as a symmetry group acting on the simplicial complex, and does that symmetry group correspond to the modular group acting on the upper half-plane?

6.5 The diagram shows 0.333 repeating and 0.666 repeating in its partial structure but the 9 — 0.999 repeating = 1 — appears only as the complete structure simultaneously. Can this be formalized as a statement about the completeness of the simplicial complex?

7. Priority Statement

This document establishes priority of authorship for the following original contributions:

(1) The XYZ coordinate framework — within/without/around — as a three-part generative decomposition corresponding to interior, star, and link operators in simplicial complex theory, with explicit correspondence to the holomorphic/non-holomorphic decomposition of harmonic Maass forms.

(2) The three-dimensional tessellated simplicial structure derived from the sixth-layer Pythagorean Tetractys as a spatial visualization of the harmonic Maass form decomposition, being the first such three-dimensional spatial visualization in the literature.

(3) The perspective-invariance claim: that cube, pyramid, hexagon, and corner-of-room views of the same structure represent different observer positions within a single unified object, encoding the framework's claim that identical structure presents as different forms to observers at different positions.

(4) The identification of the theta/mock-theta meeting point as the generative origin for further structure, and the prediction that a mock of this mock extending from that point will reveal depth in the holomorphic/non-holomorphic boundary zone.

(5) The scaling property: that the XYZ structure represents any number at any scale by coordinate magnitude adjustment, corresponding to the universality of the true-1 proof across all number bases.

The whiteboard photograph (Figure 1) constitutes the original primary document. This paper constitutes the formal record. Both are dated March 14, 2026.

Acknowledgments

Developed in extended dialogue with Claude, a large language model developed by Anthropic, PBC. The AI system's contribution as analytical instrument is acknowledged. The geometric construction, the XYZ framework, the identification of the correspondence with harmonic Maass forms, and all original claims are the author's own. The AI system's inability to be fully explained by its developers is noted as consistent with the Recursive Incomprehensibility framework developed alongside this geometric work.

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